J. Limnol., 2014; 73(3): 551-561
DOI: 10.4081/jlimnol.2014.897

ORIGINAL ARTICLE

Computing the transport time scales of a stratified lake on the basis of Tonolli's model

Marco PILOTTI,* Stefano SIMONCELLI, Giulia VALERIO

Department of Civil Engineering, Architecture, Land, Environment and Mathematics (DICATAM), University of Brescia, Via Branze 43, Brescia, Italy
*Corresponding author: marco.pilotti@unibs.it

ABSTRACT
This paper deals with a simple model to evaluate the transport time scales in thermally stratified lakes that do not necessarily completely mix on a regular annual basis. The model is based on the formalization of an idea originally proposed in Italian by Tonolli in 1964, who presented a mass balance of the water initially stored within a lake, taking into account the known seasonal evolution of its thermal structure. The numerical solution of this mass balance provides an approximation to the water age distribution for the conceptualised lake, from which an upper bound to the typical time scales widely used in limnology can be obtained. After discussing the original test case considered by Tonolli, we apply the model to Lake Iseo, a deep lake located in the North of Italy, presenting the results obtained on the basis of a 30 year series of data.

Key words: Stratified lakes, transport time scales, mass balance, Lake Iseo, Lake Maggiore.

Received: October 2013. Accepted: April 2014.

INTRODUCTION

The water renewal time of a lake (sometimes also called flushing time or turn-over time), defined as

$$T_1 = \frac{V_L}{q} \qquad \text{(eq. 1)}$$

where V_L is the volume of the whole basin and q the long-term time average discharge of water passing through the lake, is introduced as an integrative indicator of the renewal capacity of a water body (Bolin and Rodhe, 1973; Monsen *et al.*, 2002). It is also contemplated as a quality parameter in the European Water Framework Directive (European Parliament, 2000). In spite of its wide use in limnology, the ratio V_L/q completely neglects the most relevant features of the hydrodynamics of a lake. Actually, this ratio has a clear meaning only in strongly idealized situations, such as that provided by a Continuously Stirred Tank Reactors (CSTR, Levenspiel, 1999), where complete and instantaneous vertical and horizontal mixing is attained. Let us consider a CSTR of volume V_L, initially filled by a tracer (for example the water initially present in the tank, with volume V_{old}) and fed by a volumetric input q of fresh water. Assuming a steady state in the CSTR, the input is equal to the output.

The probability that a tracer particle remains inside the CSTR for a time longer than t is given by:

$$P(t) = 1 - \int_0^t \psi(\tau)\, d\tau \qquad \text{(eq. 2)}$$

with $\psi(t)$ the probability density distribution of the age of particles *within* the tank, that gives the time that has elapsed since the entrance of a particle within the CSTR. The probability that particles remain inside the reactor for a time longer than $t+dt$ is equal to the product of $P(t)$ and the probability of remaining within the CSTR during dt

$$P(t+dt) = P(t)\left(1 - \frac{q}{V_L}dt\right) \qquad \text{(eq. 3)}$$

After a first order expansion of the left hand side of eq. (3), one gets

$$\frac{dP}{dt} = -\frac{qP}{V_L} \qquad \text{(eq. 4)}$$

from which

$$P(t) = e^{-\frac{qt}{V_L}} \qquad \text{(eq. 5)}$$

Eqs. (2) and (5) provide

$$\frac{dP}{dt} = -\psi(t) = -\frac{q}{V_L}e^{-\frac{qt}{V_L}} \qquad \text{(eq. 6)}$$

from which it is straightforward to observe that in a CSTR the average value of $\psi(t)$, that is the *average age* of the tracer *within* the lake (a time scale identified in the following as T_2) has the same value of the water renewal time T_1.

From a physical point of view, the probability P can also be regarded as the volume concentration of the tracer, $C(t)=V_{old}(t)/V_L$, and its variation in time is governed by eq. (5). Accordingly, one sees from eq. (5) that when $t=T_1$, 37% (*i.e.*, e^{-1}) of the original mass is still present in the tank and the complete renewal of the mass implicit in the name of renewal time is attained only after an infinite time span.

Another important limnological time scale is provided by the distribution $\varphi(t)$ of the age of water *leaving* the lake (*i.e.*, the distribution of *residence times*). The average of $\varphi(t)$, in the following identified as T_3, is known in the literature as *residence* time. It can be shown (Bolin and Rodhe, 1973) that in steady state

$$\phi(t)=-T_1\frac{d\psi}{dt} \qquad \text{(eq. 7)}$$

so that in the case of the CSTR, being $\Psi(t)$ exponential, $\varphi=\psi$ and $T_3=T_1$. In conclusion, in a CSTR, the water renewal time, that in itself would simply quantify the general exchange characteristics of a lake, has also the meaning of *average age of lake water* and of *residence time:* $T_1=T_2=T_3$. However, due to its particular dynamics, the CSTR model provides an oversimplified representation of a real lake. In reality perfect mixing is never obtained, and accordingly one may expect that the actual transport time scales T_2 and T_3 are longer. Even if one assumes infinite horizontal mixing, as reasonable in many cases, the hypothesis of instantaneous vertical mixing would be wrong in the important case of thermally stratified lakes, where vertical mass exchanges are strongly conditioned by the timing of stratification and the depth of the thermocline. In this case, the sharp temperature transition at the thermocline protects hypolimnetic waters from surface induced mixing (Fischer *et al.*, 1979). Accordingly, the evaluation of the transport time scales of a natural lake requires a model that takes into account the thermal structure of the lake.

The investigations accomplished within recent research projects (Duwe, 2003) have pursued two different ways of evaluating the transport time scales in lakes. The first is based on the temporal and spatial distribution of tracers such as rare elements or radioactive materials, which allow tracking clearly defined water masses within the lake (Herczeg *et al.*, 1998; Michel *et al.*, 1995; Ohtsuka *et al.*, 2002). The second is based on the numerical simulation of the actual lake hydrodynamics by using a three-dimensional modeling approach Castellano *et al.*, 2010) or more simplified 1D hydrodynamic model (Rueda *et al.*, 2006). These sophisticated approaches are not devoid of drawbacks and their complexity hinders their widespread use. Therefore, it seems that there is a need for simplified approaches that, by conceptualising the most relevant mixing processes, can provide an approximation of the actual value of these time scales. An interesting idea in this direction that accounts for the simplicity of box models (Officer, 1980; Hagy *et al.*, 2000) and the features of the thermal stratification, was presented in the Italian literature by Piontelli and Tonolli (1964). Their paper is divided into two parts: the first, whose author is Tonolli, qualitatively presents a methodology that can be regarded as a simple way to estimate the water age distribution within a thermally stratified lake. In the following, we shall make reference to this model as Tonolli's model. The procedure proposed by Tonolli is interesting because, while retaining a perfect mixing in the horizontal direction, it takes into account the vertical stratification of the lake. Accordingly, this model uses the information available on morphometry, thermal structure and hydrology of the lake with a very limited computational effort. Unfortunately, the procedure was illustrated only verbally by Tonolli and this, along with its publication in Italian, did not foster its diffusion within the limnologists community.

In this paper, i) Tonolli's model is reformulated in an algorithmic way, discussing in detail its real meaning and its application to the original test case of Lake Maggiore; then ii) the model is applied to a deep Italian pre-alpine lake (Lake Iseo). The algorithm is applied using the entire available data set, discussing the validity of the approach and estimating the average long term water age distribution (eq. 2) of Lake Iseo. From this distribution, as we shall explain in the following, an upper bound to the transport time scales can be computed. The spreadsheets used to reproduce the Lake Maggiore test case are provided in order to document the computational steps of the algorithm. This file can be used to reproduce the presented results and can be easily modified to apply the procedure to other thermally stratified lakes where Tonolli's hypothesis are legitimate.

FORMALIZATION OF THE MODEL BY TONOLLI

Tonolli (in Piontelli and Tonolli, 1964) verbally presented a procedure to calculate the integrative distribution (eq. 2) of the age of the ensemble of water parcels for an oligomictic lake. The procedure can be regarded as a mass (or, more properly, volume) balance of the old water originally present within the lake, where the mixed layer changes dynamically on the basis of the lake thermal structure. It is important to observe that Tonolli used the word *residence time* for what, more appropriately (Monsen *et al.*, 2002), should be called the average age T_2 of water in the lake. He did not introduce any distinction between these terms and did not provide a statistical framework for his presentation. Thanks to the property expressed by eq. (7), it is possible to use the average age T_2 to compute the residence time T_3 of a lake.

Tonolli took into account two main phenomena which control water interchange within a lake: the thermal stratification during the limnological year and the occurrence of partial or full circulation events during the spring water overturn period (oligomixis). The method is based on the three following assumptions: i) the inflowing waters, entering from the lake tributary, are mixed homogeneously only with the mixed-layer water above the thermocline; ii) the outflowing waters come only from the mixed layer; iii) the whole lake volume is kept constant during the simulation so that the entering discharge at each time step equals the outgoing discharge from the lake.

In order to provide an algorithmic explanation of Tonolli's method, let us use the same original term *old water* (using the original symbol V_{old} introduced above) for the overall water within the lake at time $t=0$. A mass balance of V_{old} allows the computation of the concentration of the old water in the lake as a function of time. The balance is made using basic information regarding the lake morphology and dynamics: i) the volume-depth curve, ii) the time series of the thermal profile of the lake and iii) the time series of the entering discharge. The time series (ii) and (iii) are used to compute the average limnological year so that the stationariety hypothesis is fully satisfied and eq. (7) applies. We shall discuss in the following a possible extension of this hypothesis. On the basis of a step-wise discretization of the expansion of the thermocline, the limnological year k is discretized into n intervals of duration θ_i measured in days. The lake is then subdivided in a corresponding set of n layers (Figure 1A), each of volume V_j and thickness h_j, with the constraint

$$V_{lake} = \sum_{j=1}^{n} V_j \qquad \text{(eq. 8)}$$

Let $V_{old\,j}$ be the amount of *old water* contained within layer j (with initial condition, at the beginning of the simulation, $V_{old\,j}=V_j$). The surface mixed layer, whose volume at the time step i has grown by including layers as far as V_i, is given by:

$$V_{mix\,i} = \sum_{j=1}^{i} V_j \qquad \text{(eq. 9)}$$

Neglecting precipitation and evaporation, the volume of water that enters and flows out of the lake during the time step i, $V_{out,i}$, is obtained by multiplying the time average discharge Q_i by the corresponding duration θ_i, so that $V_{out\,i}=Q_i \cdot \theta_i$ (Figure 1B). It is then possible to define an explicit approximation of the volume concentration c_i of the *old water* in the mixed layer at the end of the period i, as

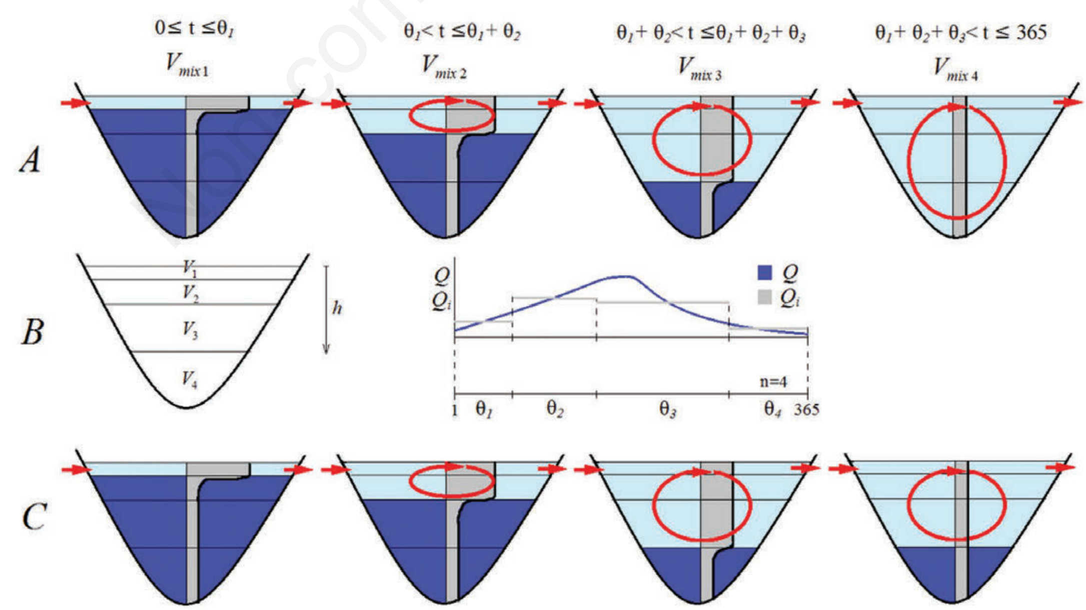

Fig. 1. Schematization of the Tonolli's model. On the basis of the observation of the mixed layer expansion in time of a monomictic lake (*A*), the lake is discretised (*B*, on the left) in a stacked set of layers, with volumes V_i whose thickness follows the expansion of the mixed layer during the interval θ_i over the limnological year; here i=4. The average discharge Q_i from the tributary is computed by averaging the inflowing discharge Q over each interval (*B*, on the right). In the case of a meromictic lake (*C*), the deepest layer is not involved in the mixing.

$$c_i = \frac{\sum_{j=1}^{i} V_{old\ j}}{V_{mix\ i} + V_{out\ i}} \quad \text{(eq. 10)}$$

which represents the ratio between the *old water* available in the mixed layer at the beginning of the time step i and the virtual water volume available in the same time interval, obtained as a sum between the mixed layer ($V_{mix\ i}$) and the inflowing water ($V_{out,i}$) volumes. The lower case is here used in order to distinguish the volume concentration in the mixed layer from the overall volume concentration in the lake, $C(t)$. The ratio (eq. 10) can also be regarded as the probability to draw an *old water particle* from the mixed layer during period i, with $i \geq 1$. The concentration (eq. 10) is used to update the final volume of *old water* within each layer j that has been included in the mixed layer:

$$V_{old\ j} = c_i \cdot V_j \quad \text{(eq. 11)}$$

This simple procedure can be applied to the next time period θ_{i+1} expanding the mixed layer up to V_{i+1} and computing the new initial conditions for $V_{old\ j}$.

The explained algorithm redistributes *old water* from the deeper layers towards the surface, increasing the *old water* concentration in the new mixed layer after its expansion. The iteration of this algorithm to the n periods, in which the year has been subdivided, yields the old water volume lost from the lake after one year. For instance, at the end of the first year this volume is $\left(V_{lake} - \sum_{j=1}^{n} V_{old\ j}\right)$, or, in relative terms with respect the overall lake volume

$$\beta_1 = \frac{\left(V_{lake} - \sum_{j=1}^{n} V_{old\ j}\right)}{V_{lake}} = 1 - \frac{\sum_{j=1}^{n} V_{old\ j}}{V_{lake}} \quad \text{(eq. 12)}$$

where the symbol β introduced by Tonolli has been retained. The same quantity can be easily generalized for the end of a year k >1, as

$$\beta_k = 1 - \frac{\sum_{j=1}^{n} V_{old\ j,k}}{V_{lake}} \quad \text{(eq. 13)}$$

where the old water volume is referred to the end of the year k.

The described algorithm can be used to reproduce the behaviour of monomictic, oligomictic or even meromictic lakes. In the first case, $V_{mix\ n}$ encompasses the whole volume of the lake, V_{lake}. In the meromictic case, relation (9) must be redefined as

$$V_{mix\ i} = \begin{cases} \sum_{j=1}^{i} V_j & i < n \\ \sum_{j=1}^{n-1} V_j & i = n \end{cases} \quad \text{(eq. 14)}$$

so that $V_{mix\ n} < V_{lake}$ because the deepest layer n is not involved in the mixing layer expansion and its old water content is left untouched (see case C of Figure 1). The oligomictic case is not uncommon, particularly in deep pre-alpine lakes, where the analysis of long time series of temperature and dissolved oxygen often shows that a full circulation event occurs only episodically, with an average periodicity τ. For instance, considering some European prealpine lakes, in the 30 years period between 1980 and 2010, Lake Garda and Lake Constance fully mixed on average once every 3 years, Lake Geneva once every 4 years and Lake Iseo once every 7 years. An oligomictic behaviour is reproduced when one year of monomictic behaviour follows τ years of meromictic behaviour.

As observed above, Tonolli's method can be seen as a mass balance of the old water contained within a CSTR whose volume changes dynamically with time on the basis of the lake thermal stratification. To this purpose, it can be observed that the eqs. (10) and (11), used to update the old water content within each layer, can be seen as an implicit finite difference first order approximation of eq. (4)

$$\frac{V_{old}(t+\Delta t) - V_{old}(t)}{\Delta t} = -\frac{qV_{old}(t+\Delta t)}{V_L} \quad \text{(eq. 15)}$$

The quantity 1- β_k is the old water concentration $C(t_k)$ (see eq. 2, where P has the same role of C) at the end of each *k-th* year. As shown by (eq. 2), β_k can also be regarded as the cumulative distribution of the age of the *old water* in the lake. Accordingly, the derivative of β_k provides the *old water* age probability density distribution $\psi(t)$, from which the average water age T_2 in the lake

$$T_2 = \int_0^\infty t\psi(t)dt \quad \text{(eq. 16)}$$

can be computed.

If one makes the assumption that the hydrological and thermal forcings of the lake are periodic, the process is stationary and eq. (7) can be applied to compute the distribution $\varphi(t)$, of residence times from $\psi(t)$. The periodicity can be at an yearly scale (case of monomictic lake) or on a longer time scale (oligomictic case). In both cases, given that the solution of equation (4) is exponential, Tonolli's method provides a water age distribution $\psi(t)$ that can be locally approximated by an exponential pattern (as will be evident considering the following test cases). Accordingly, eq. (7) provides a simple situation where the two probability distributions φ and ψ tends to be the same and also the corresponding time scales (T_2 and T_3), which in the following we shall define as $\overline{T}$, are the same. As will be shown in the following, the ratio $\overline{T}/T_1$ can be used to quantify the gap between the perfect mixing of a CSTR and the actual level of mixing of a real lake.

THE LAKE MAGGIORE CASE STUDY

Tonolli applied his model to the case of Lake Maggiore, subdividing the year into 6 time intervals. He considered a periodic sequence of average limnological years. In the following we will repeat in detail his computations using the symbols introduced above.

Lake Maggiore is located in the Italian pre-alpine area, has a large volume (37.7 km^3) and a considerable maximum (370 m) and mean depth (177 m). Due to the particular climatic zone where this lake is located, in Lake Maggiore the winter circulation does not succeed in mixing each year the whole mass of water down to the greatest depth but is frequently restricted to the first 100 to 200 meters. In the period investigated by Tonolli this lake was, on average, characterized by a cycle of 5 years of partial water overturn followed by a complete homogenization. On the basis of the data provided by Tonolli, the theoretical water renewal time T_1 of lake Maggiore is 37,700/9406≃4 years, where the volume of the lake (37,700) and the average yearly volume (9406) that flows into the lake are measured in millions of cubic meters.

In order to show in detail the application of the algorithm, Tab. 1 shows the quantities that are needed to use Tonolli's model and Tab. 2 shows the detailed computations for two out of three of the possible cases: the monomictic and the meromictic one, representing the theoretical behaviour and the limiting degenerative situation that could affect this lake. A similar table for the oligomictic case is provided as a Supplementary Excel file. Fig. 2 shows the concentration of V_{old} as a function of time in Lake Maggiore for the oligomictic (curve 1), monomictic (curve 2) and meromictic (curve 3) cases. Assuming the oligomictic behaviour that has characterized this lake over the last decades and using a time step of 1 day, the computed value of $\overline{T}$ would be 13.2 years ($\overline{T}/T_1$=3.3). In case of a monomictic behaviour, the value of $\overline{T}$ would be of 10.7 years ($\overline{T}/T_1$=2.67). By increasing the time step up to 30 days, due to round-off errors one can expect a variation of the results in the order of 10%.

THE LAKE ISEO CASE STUDY

To our knowledge, the Tonolli's algorithm was applied by Buzzi *et al.* (1997) to Lake Como, the deepest Italian pre-alpine lake, located 50 km east of Lake Maggiore. Since this lake has some thermal similarities with Lake Maggiore, Buzzi assumed the same mixing depths series of Lake Maggiore but no details of the computations were provided. In the following, we show the application of Tonolli's procedure to Lake Iseo for the period between 1977 and 2012, where a wide dataset is available to characterize the main hydrodynamic features of the lake (Pilotti *et al.* 2013). In particular, a detailed observation of the data of the tributaries for this deep and large prealpine lake may help to identify the relevance of one of Tonolli's hypothesis and to better pinpoint the field of application of this simple methodology.

Lake Iseo is located in the subalpine area of Italy (Fig. 3) and it is the fourth largest Italian lake with a volume of about 7.9·10^9 m^3, an area of 60.88 km^2 and a maximum depth of about 256 m in the central part of the basin (S1 in Fig. 3). It is fed by two main tributaries, the River Oglio (I1 in Fig. 3) and the Canale Industriale (I2) which is a diversion of I1 used for hydropower energy production. The average discharge of these tributaries is the same and is about 27 m^3/s; however in the last 10 years I1 has registered a maximum average daily discharge of 600 m^3/s with respect to 50 m^3/s of I2. A two years long continuous temperature monitoring is available at I1 and I2. The lake level and outflow are controlled by a low dam at Sarnico

Tab. 1. Data for the Lake Maggiore test case. The average periodicity of full circulation is 5 years. During the last year of each period, the mixed layer expansion 7B must be considered in place of 7A, that characterizes the partial circulation of ordinary years. h_i is the thickness of the mixed layer at time θ_i.

i	Period (day, month)	θ_i (days)	h_i (m)	V_j (10^6 m^3)	$V_{mix\ i}$ (10^6 m^3)	Q_i (10^6 m^3/day)	$V_{out\ i}$ (10^6 m^3)
1	1/4-16/11	229	0-10	2080	2080	31.8	7282
2	17/11-15/12	31	0-20	1990	4070	23.22	720
3	16/12-31/12	15	0-30	1910	5980	17.90	269
4	1/1-16/1	16	0-40	1810	7790	12.50	200
5	17/1-2/3	45	0-50	1710	9500	12.62	568
6	3/3-18/3	15	0-100	7900	17400	12.40	186
7A	19/3-31/3	14	0-100	7900	17400	12.40	174
7B	19/3-31/3	14	0-370	20300	37300	12.30	173

Tab. 2. Detailed computation for Lake Maggiore for the meromictic and monomictic cases. A detailed computation is performed for the first year and then only the final results are shown for the following 4 years. A complete list for 30 years is available in the Excel spreadsheet provided as Supplementary file.

Year k	i	θ_i (days)	h_i (m)	V_j (10^6 m^3)	$V_{mix\ i}$ (10^6 m^3)	Q_i (10^6 m^3/days)	$V_{out\ i}$ (10^6 m^3)	$V_{old\ j}$ (10^6 m^3)	c_i	$C(t_k)$=1-β_k
Meromictic case										
1	1	229 10	2080	2080	31.80	7282	2080	0.222	0.920	
	2	31	10	2080	4070	23.22	720	462	0.512	
			20	1990				1990		
	3	15	10	2080	5980	17.90	269	1065	0.639	
			20	1990				1019		
			30	1910				1910		
	4	16	10	2080	7790	12.50	200	1329	0.705	
			20	1990				1272		
			30	1910				1221		
			40	1810				1810		
	5	45	10	2080	9500	12.62	568	1466	0.715	
			20	1990				1403		
			30	1910				1346		
			40	1810				1276		
			50	1710				1710		
	6	15	10	2080	17400	12.40	186	1488	0.836	
			20	1990				1423		
			30	1910				1366		
			40	1810				1295		
			50	1710				1223		
			100	7900				7900		
	7A	14	10	2080	17400	12.40	174	1738	0.827	
			20	1990				1663		
			30	1910				1596		
			40	1810				1512		
			50	1710				1429		
			100	7900				6601		
2										0.854
3										0.800
4										0.755
5										0.717
Monomictic case										
1	1	229	10	2080	2080	31.8	7282	2080	0.222	0.920
	2	31	10	2080	4070	23.22	720	462	0.512	
			20	1990				1990		
	3	15	10	2080	5980	17.9	269	1065	0.639	
			20	1990				1019		
			30	1910				1910		
	4	16	10	2080	7790	12.5	200	1329	0.705	
			20	1990				1272		
			30	1910				1221		
			40	1810				1810		
	5	45	10	2080	9500	12.62	568	1466	0.715	
			20	1990				1403		
			30	1910				1346		
			40	1810				1276		
			50	1710				1710		
	6	15	10	2080	17400	12.4	186	1488	0.836	
			20	1990				1423		
			30	1910				1366		
			40	1810				1295		
			50	1710				1223		
			100	7900				7900		
	7B	14	10	2080	37700	12.4	174	1738	0.920	
			20	1990				1663		
			30	1910				1596		
			40	1810				1512		
			50	1710				1429		
			100	7900				6601		
			370	20300				20300		
			20	1990				1407		
			30	1910				1351		
			40	1810				1280		
			50	1710				1209		
			100	7900				5586		
			370	20300				17178		
2										0.846
3										0.778
4										0.716
5										0.659

(O1 in Fig. 3) for agricultural purposes. The daily water surface variations are negligible with a maximum of 1.4 m over the year. During most of the year the lake presents a strong thermal stratification with bottom water temperature around 6.4°C (Fig. 4); even though the lake can be classified as warm monomictic according to its geographical location, the full circulation events only happen sporadically during very cold and windy winters. Accordingly the lake is oligomictic with an anoxic condition at the bottom (Ambrosetti and Barbanti, 2005) and with only 5 full circulations over the 1977-2012 period (filled circles in Fig. 7). Over the last 20 years the main physical and

Tab. 3. Data for the Lake Iseo test case. The average periodicity of full circulation is 7 years. During the last year of each period, the mixed layer expansion 351B must be considered in place of 351A, that characterises the partial circulation of ordinary years. h_i is the thickness of the mixed layer at time θ_i.

i	Period (day, month)	θ_i (days)	h_i (m)	V_j (10^6 m^3)	$V_{mix\ i}$ (10^6 m^3)	Q_i (10^6 m^3/day)	Q_i (m^3/s)	$V_{out\ i}$ (10^6 m^3)
1	10/4-15/6	98	0-6	354	354	6.44	74.50	630.8
99	16/6-20/8	66	0-10	226	580	6.10	70.60	402.6
165	21/8-25/10	66	0-17	333	913	6.05	70	399.2
231	26/10-26/12	62	0-36	1028	1941	8.73	101	671.9
293	27/12-7/2	43	0-78	1905	3846	4.58	53	125.80
336	8/2-22/2	15	0-100	817	4663	4.06	47	112.75
351A	23/2-9/3	15	0-100	817	4663	3.54	41	53.14
351B	23/2-9/3	15	0-260	3242	7905	3.54	41	53.14

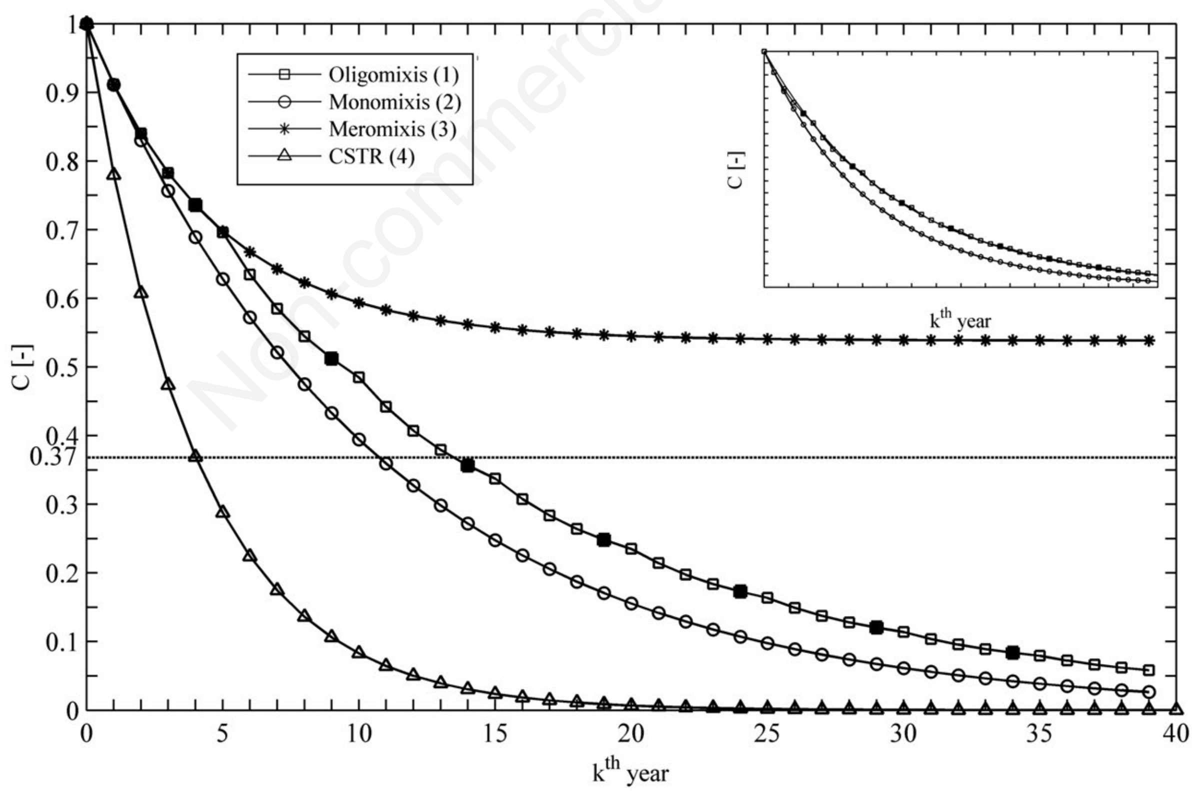

Fig. 2. Old water concentration *C(t)* as a function of time for Lake Maggiore. Four different situations are shown: the monomictic condition (2), the meromictic condition (3), with a mixed layer of 100 m, and the oligomictic one (1), with alternating years of partial and complete circulation events. On the oligomictic curve the full circulation years are shown by filled squares. The CSTR (4) case, parameterized in terms of the theoretical renewal time, is shown as a comparison. The inset in the upper right corner shows the match between the computed data for case (1) and (2) and the exponentials with time scale given by the computed $\overline{T}$ values.

chemical parameters of water have been monitored monthly along a vertical profile at one of the deepest locations of the lake (Garibaldi *et al.*, 1999). Since 2010 we have carefully monitored the vertical thermal structure of the lake using a thermistor chain tied at the floating station moored a few km in front of the tributary entrances (LDS in Fig. 3), as well as the temperature of the tributaries (Fig. 6). The chain has 21 sensors located at a depth from 0 to 50 m and it measures the data in real time. A full characterization of the data set available for Lake Iseo is provided in Pilotti *et al*. (2013).

These data were used to derive the temporal evolution of the mixing depth in the period 1977-2012. The yearly dynamic is characterized by a gradual increase of the mixing depth between March and mid-October i), followed by a faster deepening in winter ii), which ends at the beginning of March, when the maximum mixing extent is reached. This extent is different year by year, depending on the winter meteorological conditions. With regard to phase (i), the high resolution temperature data measured by the thermistor chain at LDS station made it possible to compute the Brunt-Väisälä frequency *N*; the mixing depth was assumed to be placed in correspondence of the maximum value of this parameter. Because of the limitation of the thermistor chain length, it was not possible to obtain the complete evolution of the mixing depth with this analysis. Moreover, these data were limited to three limnological years only (2010-2012). Since the time series of the maximum values of *N* showed a similar trend for the three years, it was regarded as valid also for the remaining limnological years. With regard to the phase (ii), we assumed a linear increase of the mixing depth down to the maximum values observed in winter. In order to obtain the series of the maximum mixing depth for the period 1977-2012 the following data were also used: i) the dissolved oxygen concentration profiles as a function of time from 1984-2009 (Garibaldi *et al.*, 1997 and Mosello *et al.*, 1997): these profiles were used to evaluate the maximum extent of mixing at the end of the spring water over-

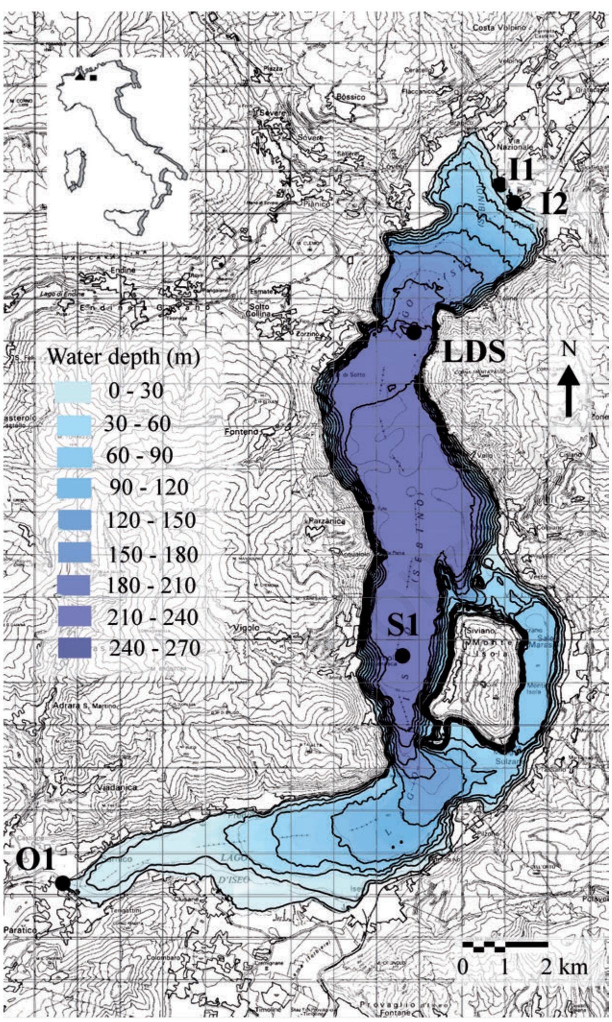

Fig. 3. Geographical setting of Lake Maggiore and Lake Iseo (respectively, triangle and square in the upper left inset), along with the Iseo bathymetry, represented by isodepth lines at 30-m spacing. The location of the two main tributaries is shown as I1 and I2. The bottom left corner coordinates are WGS84 573067 E, 5054053 N.

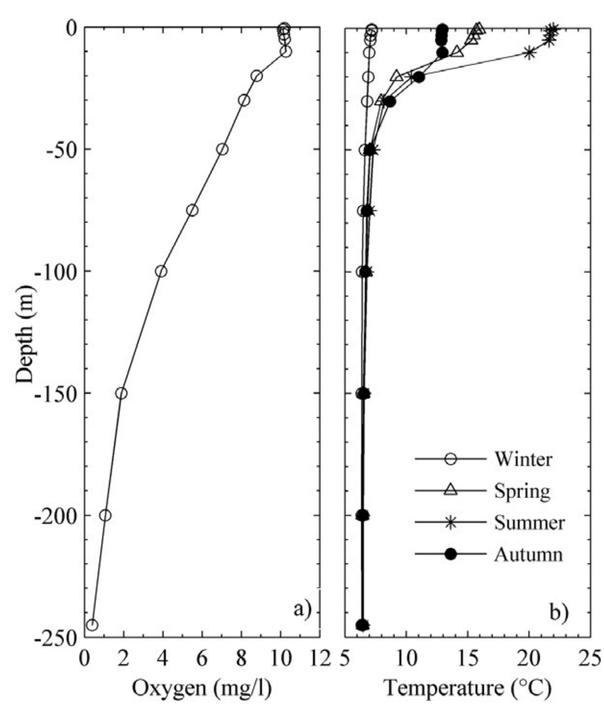

Fig. 4. Average vertical profiles of a) oxygen and b) temperature in Lake Iseo computed on the basis of the 1995-2010 time series (Pilotti *et al.*, 2013).

turn (mid-March) on the basis of the oxygen variations in the mesolimnic and hypolimnic layers; ii) the maximum mixing depths for the period 1978-1983 provided by Ambrosetti and Barbanti (2005). The complete time series of mixing depths is shown as a dashed line in Fig. 7: from this series it is possible to observe that over the last 35 years, 5 complete winter circulations were observed, with an average maximum depth of winter circulation during the remaining years of about 100 m (computed as a volumetric average).

Accordingly, in the following we applied the revised Tonolli's model to an average oligomictic sequence characterized by 6 years of partial water overturn followed by a complete homogenization, characterized as in Tab. 3. Fig. 5 shows the computed old water concentration *C(t)* within the lake under the effect of the average oligomictic sequence. The computation was accomplished using a daily time step. The value of the computed average of *C(t)*, $\overline{T}$, is 8.8 years and has to be compared with the theoretical CSTR value T_I=4.5 years ($\overline{T}$ /T_I=1.95). It might be interesting to test the validity of the Tonolli's hypothesis regarding the entrance of the tributary rivers within the mixed layer. Depending on the density difference between the lake and the tributary waters, either overflow, interflow or plunging flow may occur (Fischer *et al.*, 1979). Disregarding the suspended sediment transported by the River Oglio, which has never been measured but which might be relevant during floods, we shall consider the effect of the temperature difference only. Fig. 6 shows the difference between the measured average epilimnic

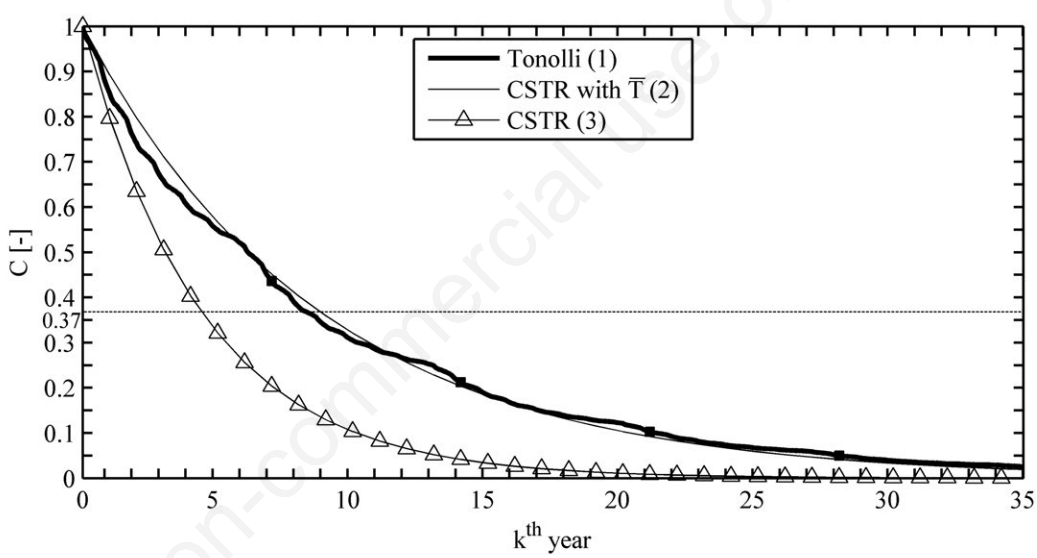

Fig. 5. Computed old water concentration *C(t)* for Lake Iseo, superimposed to the exponential pattern parameterized with $\overline{T}$=8.8 years. The filled squares highlight the full circulation events. The CSTR pattern parameterized with T_I is shown as a comparison.

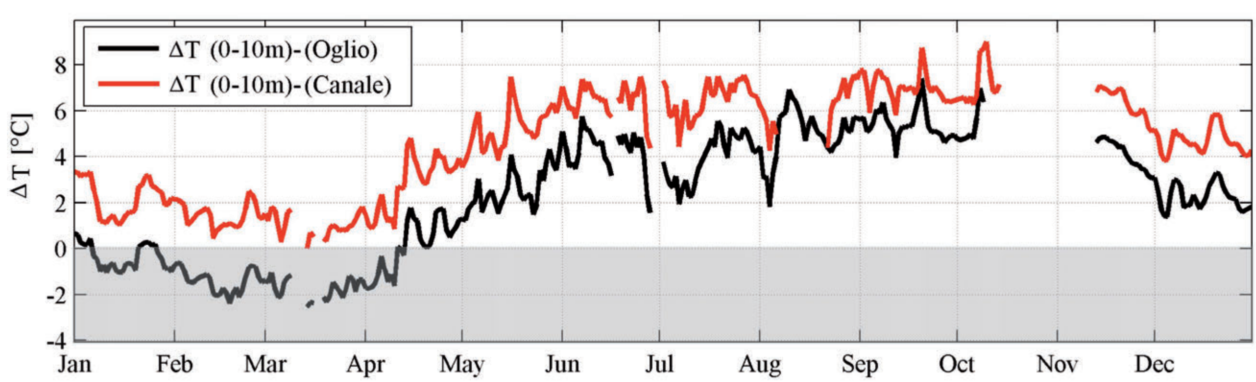

Fig. 6. Difference between the averaged water temperature in the first 10 m of the water column and the temperature of the tributary rivers in I1 (Oglio) and I2 (Canale); the measurement period is 2011.

temperature in the first 10 m of the water column, where the thermocline is located during most of the year, and the temperature of the tributaries, as a function of time. The figure shows that interflow is a dominant process all year long in Lake Iseo. It is not surprising that the tributary rivers have typical temperatures significantly lower than those of the epilimnion of the lake because it is located at an elevation of 168 m asl, and drains a large Alpine catchment, Valle Camonica, whose average and maximum altitudes are respectively 1401 m asl and 3554 m asl. Accordingly, the hypothesis of Tonolli is not totally legitimated in the case of this Italian prealpine lake. Considering that interflow enhances mixing, Tonolli's method presumably provides an upper bound to the *actual* value of the average residence time, that must lie in the range between T_1 (4.5 years) and $\overline{T}$ (8.8 years).

As a final consideration, one can observe that, in place of using the average series of Tab. 3, the model could directly work with the measured chronological time series. Fig. 7 shows the old water concentration $C(t)$ computed using the time series of the outflowing discharges and mixing depths for the 1977-2012 period. In this case, as one can observe, the strong variability of the maximum mixing depth causes a significative oscillation of the $C(t)$ curve around the exponential pattern computed on the basis of the average data of Tab. 3. This is a consequence of the natural variability of the process: if one assumes that each year is statistically independent, the 1977-2012 sequence is only one among the possible realizations of a stochastic process of a series of 35 years with 5 complete circulation events.

CONCLUSIONS

In this paper we presented an algorithmic explanation of a simple model to compute the distribution $\psi(t)$ of the age of water within a thermally stratified natural lake, based on an idea proposed by Tonolli (Piontelli and Tonolli, 1964). This model can be easily applied whenever a sufficiently long time series of the thermal structure and of the tributary data are available or can be obtained by an hydrologic similitude with similar lakes in the same geographic area. On the basis of a stationary hypothesis we show that it is possible to derive the probability distribution $\varphi(t)$ of residence time of water. Accordingly, the methodology allows an easy evaluation of the average age of water, T_2, and of the residence time, T_3, within a lake. These time scales are extremely important for understanding the relationship between the chemistry of the original tributary waters and that of the waters within the lake, as well as the sensitivity of the lake to direct pollution and in Tonolli's model have the same value $\overline{T}$.

In most limnological classification of lakes, the consid-

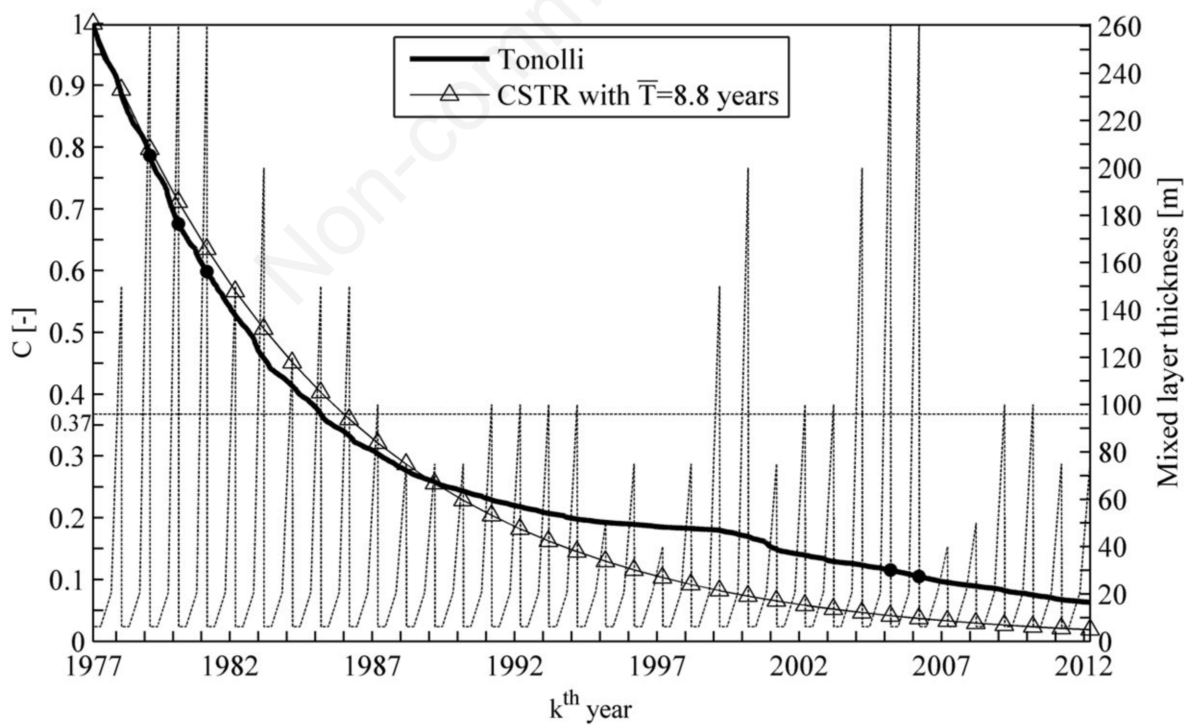

Fig. 7. Old water concentration $C(t)$ within Lake Iseo starting from 1977 (solid line): the filled dots highlight the years when full circulation events occurred, as shown also by the dashed line that is the measured time series of the mixing depth. The lower line is the exponential curve for a CSTR parameterized with the computed value $\overline{T}$=8.8 years.

ered time scales are assumed equal to the theoretical water renewal time, T_1, with implicit reference to a continuous stirred tank reactor (*CSTR*) where this scale also corresponds to the time when only 37% of the original water (or of a passive tracer dissolved within it) is still present within the lake. However, due to the perfect mixing hypothesis, T_1 is rather the lower bound of the actual value of the average residence time, T_3, and the ratio T_3 / T_1 can be considerably greater than 1 for strongly stratified lakes, especially when complete winter circulation does not occur regularly. Accordingly, if T_1 provides a lower bound to T_3, it seems important to find an upper bound for this quantity, taking into account some of the most relevant aspects of the complex hydrodynamics of a lake.

The model of Tonolli is conceptually straightforward and provides this approximation. It makes direct use of the measured thermal structure of the lake, that in itself reflects the mixing effect of possible internal waves acting in the lake. However, it has an evident limitation since it assumes that the inflowing water from the lake tributary enters in the mixed layer only. This limitation is discussed with reference to the data collected over the last 3 years in Lake Iseo, highlighting that in this lake the hypothesis is not satisfied for most part of the year - both in winter and in summer season. In such a case, due to density differences, the entering waters could originate an interflow deeper than the mixed epilimnic layer or even a plunging flow, sinking to the lower layers of the lake. Both these processes stimulate further mixing and contribute to a decrease in the water age. Accordingly, Tonolli's hypothesis is strictly true when either overflow or shallow interflow occurs in a lake, *i.e.* when the tributary waters are not denser than the thermocline waters of the lake. In such a case, the value that can be computed by this methodology closely reproduces the most relevant mixing mechanism. In the other cases, however, the value provided by Tonolli's method can be regarded as an upper bound to T_3, contributing, to limit the range of uncertainty of this characteristic time. The results for Lake Maggiore, a deep thermally stratified lake in the Italian prealpine area, emphasize the consequences in terms of water age of different mixing regimes: oligomictic ($\overline{T}$=13.2 years; $\overline{T}/T_{1=}$3.3), monomictic ($\overline{T}$=10.7; $\overline{T}/T_1$=2.67) and CSTR behaviour ($\overline{T}$=4 years; $\overline{T}/T_1$=1). The application to the Lake Iseo test case, the fourth largest Italian lake in the same prealpine area, provides a similar computed ratio $\overline{T}/T_{1=}$1.95.

The algorithm is implemented for lake Maggiore in a simple spreadsheet whose structure can be easily adapted to that of other thermally stratified lakes. The file is provided as a Supplementary Excel file.

ACKNOWLEDGMENTS

We acknowledge the anonymous Reviewers for their valuable contributions to the improvement of this paper.

REFERENCES

Ambrosetti W, Barbanti L, 2005. Evolution towards meromixis of Lake Iseo (Northern Italy) as revealed by its stability trend. J. Limnol. 64:1-11.

Bolin B, Rodhe H, 1973. A Note on the concepts of age distribution and transit time in natural reservoirs. Tellus 25:58-62.

Buzzi F, Gerosa G, Saldavè G, 1997. Descrizione ed analisi di alcuni aspetti limnologici e idrodinamici del lago di Como. Documenta Ist. Ital. Idrobiol. 61: pp. 93-115.

Castellano L, Ambrosetti W, Barbanti L, Rolla A, 2010. The residence time of the water in Lago Maggiore (N. Italy): first results from an Eulerian-Lagrangian approach. J. Limnol. 69:15-28.

Duwe K, 2003. D24: realistic residence times studies integrated water resource management for important deep European lakes and their catchment areas. EUROLAKES FP5 Contract No. EVK1-CT1999-00004.

European Parliament, 2000. Directive 2000/60/EC of the European Parliament and of the Council of 23 October 2000 establishing a framework for Community action in the field of water policy.

Fischer HB, List EJ, Koh RCY, Imberger J, Brooks NH, 1979. Mixing in inland and coastal waters. Academic Press.

Garibaldi L, Brizzio MC, Mezzanotte V, Varallo A, Mosello R, 1997. [Evoluzione idrochimica e trofica del Lago d'Iseo].[Article in Italian]. Documenta Ist. Ital. Idrobiol. 61:135-151.

Garibaldi L, Mezzanotte V, Brizzio MC, Rogora M, Mosello R, 1999. The trophic evolution of Lake Iseo as related to its holomixis. J. Limnol. 58:10-19.

Hagy JD, Boynton WR, Sanford LP, 2000. Estimation of net physical transport and hydraulic residence times for a coastal plain estuary using box models. Estuaries 23:28-340.

Herczeg AL, Imboden D, 1988. Tritium hydrologic studies in four closed-basin lakes in the Great Basin, U.S.A. Limnol. Oceanogr. 33:157-173.

Levenspiel O, 1999. Chemical reaction engineering. 3. J. Wiley & Sons.

Michel RL, Kraemer TF, 1995. Use of isotopic data to estimate water residence times of the Finger Lakes, New York. J. Hydrol. 164:1-18.

Monsen NE, Cloern JE, Lucas LV, Monismith SG, 2002. A comment on the use of flushing time, residence time, and age as transport time scales. Limnol. Oceanogr. 47:1545-1553.

Mosello R, Calderoni A, De Bernardi R, 1997. [Le indagini sulla evoluzione dei laghi profondi subalpini svolte dal CNR].[Article in Italian]. Ist. Ital. Idrobiol. 61:19-32.

Officer CB, 1980. Box models revisited, p. 65-114. In P. Hamilton and K.B. MacDonald (eds.), Estuarine and wetland processes with emphasis on modeling. Plenum Press, New York.

Ohtsuka Y, Yamamoto M, Sasaki K, Komura K, 2002. Cosmogenic radionuclide ^{22}Na as an index in evaluating residence time of lake water. Radioprotection. 37:C1-63, C1-68.

Pilotti M, Valerio G, Leoni B, 2013. Data set for hydrodynamic lake model calibration: a deep pre-alpine case. Water Resour. Res. 49:7159-7163.

Piontelli R, Tonolli V, 1964. [Il tempo di residenza delle acque lacustri in relazione ai fenomeni di arricchimento in sostanze immesse, con particolare riguardo al Lago Maggiore].[Article in Italian]. Mem. Ist. Ital. Idrobiol. 17:247-266.

Rueda F, Moreno-Ostos E, Armengol J, 2006. The residence time of river water in reservoirs. Ecol. Modell. 191:260-274.